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GUIDE06 GUIDELINES FOR CALCULATING GRAVITY POTENTIAL VALUES AT IHRF STATIONS IN THE SIRGAS REGION

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DOCUMENT CHANGE LOG

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This document has been structured based on meetings of the members of WG III and the bibliographic references cited in the "References" section.

In order to keep this document up to date, we cordially invite you to send your comments, questions, or suggestions to the Chair of SIRGAS-GTIII, whose contact information can be found at https://sirgas.ipgh.org/.

GENERAL INFORMATION

In 2015, the International Association of Geodesy (IAG) published Resolution No. 1, which deals with defining and implementing an International Height Reference System (IHRS). The IHRF (International Height Reference Frame) will be based on the materialization and implementation of a set of previously selected stations.

This document presents the formulations, constants, and conventions for calculating gravity potential values at an IHRF station using regional gravity field models (geoids and pure gravimetric quasi-geoids). These recommendations and guidelines are based on IHRF Coordination Center (IHRF CC) documents, "IHRF Conventions – Simplified" and "Recovering Potential Values from Regional (quasi-)geoid Models," which are available on the IHRF[1] CC website and in the IGM Uruguay document (IGM, 2024). The document concludes with examples of numerical values that interested parties can use to calculate gravity potential values with their own tools and programs, and then compare these values with those calculated by the WG-III.

Those interested in calculating gravity potential values at IHRF stations should ensure that their input coordinates are official and provided by the IHRF CC through SIRGAS. For practical applications, Cartesian coordinates (X, Y, Z) must be transformed to ellipsoidal coordinates (φ, λ, h) using the 1980 Geodetic Reference System (GRS80). The geodetic latitude and longitude coordinates should be expressed with eight decimal places, and the ellipsoidal height with three. Additionally, it is important to know whether the gravity value at the station is observed or interpolated. If the value is observed, it is crucial to know exactly where the measurement was made and what type of observation it is.

For a correct calculation of the gravity potential, it is important to know how the pure gravimetric geoid or quasi-geoid model was calculated. It is necessary to be clear about the concept of permanent tides of coefficient $C_{2,0}$ of the global geopotential model and whether the first part of the zero-order term of equations (8) and (9) was taken into account when using the global geopotential model.

It is also important to know whether the geoid or quasi-geoid models satisfy the condition $W_0=U_0$ or W_0 different from U_0, in order to be clear whether the second term of equations (8) and (9) was taken into account.

If in doubt, it is recommended to contact the agency or university that calculated the geoid or quasi-geoid model.

I. CONSTANTS AND CONVENTIONS

The vertical coordinate at a point *P* is the difference between the gravitational potential at that point, W_P, and the reference potential value W_0. This difference is known as the geopotential number C_P:

[1] https://ihrfcc.topo.auth.gr/

$$C_P = -\Delta W_P = W_0 - W_P \qquad (1)$$

where the value of W_0 is 62,636,853.4 m^2s^{-2} (Drewes et al. 2016).

The spatial position of *P* for the potential is given by the coordinates of vector X_P in the ITRS (International Terrestrial Reference System), such that: $X_P = X(P)$ *and* $W(P) = W(X_P)$. All parameters, observations, and data must be in terms of mean tide (*mean tidal/mean crust*), which eliminates the temporal and periodic component of the tidal potential and preserves the permanent component and the potential generated by the deformation of that component. The units of length and time are respectively the meter, *m*, and the second, *s*, given in SI (International System of Units). The Geodetic Reference System is GRS80 and the parameters are presented in Table 1. All parameters defined by GRS80 must be used as defined, without performing any prior calculations to determine them.

Table 1 GRS80 parameters

Parameters	**Value**	**Unit**	**Description**
a	6 378 137,0	*m*	major semi-axis of the ellipsoid
b	6 356 752,3141	*m*	minor semi-axis of the ellipsoid
e^2	0,00669438002290	-	first eccentricity of the ellipsoid
U_0	6 263 686,0850 x 10	$m^2 s^{-2}$	normal gravitational potential
γ_e	9,7803267715	$m^2 s^{-2}$	normal gravity at the equator
GM	3,986004415 x 10^{14}	$m^3 s^{-2}$	geocentric gravitational constant
f	0,00335281068118	-	flattening
m	0,00344978600308	-	$\frac{\omega^2 a^2 b}{GM}$

Figure 1 (Sánchez et al. 2021) details the relationships between the various reference surfaces, heights, and points involved in the formulations that will be presented.

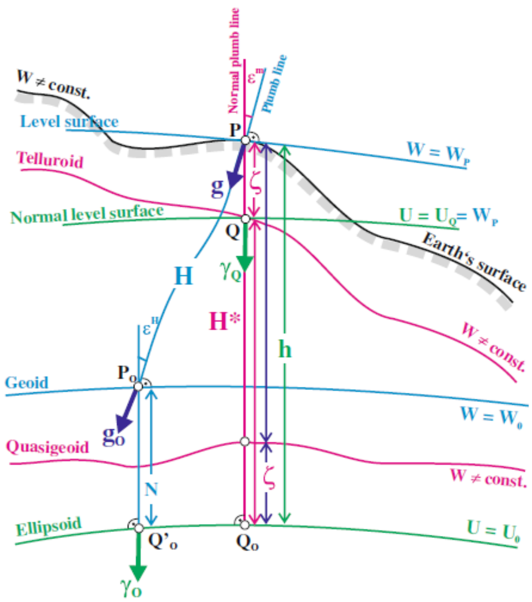

Figure 1 Relationships between different reference surfaces, heights, and points for calculation (Sánchez et al. 2021).

2. FORMULAS

The closed equation of theoretical gravity is presented according to (Moritz 2000):

$$\gamma_0 = \frac{a\gamma_a \cos^2 \varphi + b\gamma_b \sin^2 \varphi}{\sqrt{a^2 \cos^2 \varphi + b^2 \sin^2 \varphi}}, \qquad (2)$$

where *a* and *b* are the semi-major axis and semi-minor axis of the ellipsoid.

The geocentric radial distance r_P is calculated as (Tovar Cabañas et al. 2023):

$$r = \frac{ab}{\sqrt{(a \cdot sen\psi)^2 + (b \cdot cos\psi)^2}} + h \qquad (3)$$

where ψ is the geocentric latitude equal to $\psi = arctan[(1 - e^2) \cdot tg\varphi]$, φ is the geodetic latitude, *a* and *b* are the major and minor semi-axes of the ellipsoid, and e^2 is the first eccentricity. The geocentric radial distance at *P* is $r_P = r + h$, where *h* is the ellipsoidal height.

2.1 EQUATIONS FOR CALCULATING GRAVITY POTENTIALS FROM PURE GRAVIMETRIC MODELS OF THE QUASI-GEOID AND GEOID

Based on a quasi-geoid model, the value of the gravitational potential at point *P* on the Earth's surface can be obtained using:

$$W_P = W_0 - (h_P - \zeta_P) \cdot \overline{\gamma}_{QQ_0} [m^2 s^{-2}] \qquad (4)$$

where h_P is the ellipsoidal height of the IHRF station, ζ_P is the interpolated height anomaly of the pure quasi-geoid model. Where: $\overline{\gamma}_{QQ_0}$

$$\overline{\gamma}_{QQ_0} = \gamma_0 \cdot \left(1 - \frac{1}{a} \cdot (1 + f + m - 2f \cdot sen^2\varphi_P) \cdot (h_P - \zeta_P)\right) [m^2 s^{-2}] \qquad (5)$$

where $\gamma_0$is the normal gravity on the reference ellipsoid calculated using equation (2) and $\varphi_P$is the geodetic latitude of the IHRF station.

For a pure gravimetric geoid model, the gravity potential at point *P* is obtained using the formula

$$W_P = W_0 - (h_P - N_P) \cdot \overline{g}_P [m^2 s^{-2}] \qquad (6)$$

being

$$\overline{g}_P = g_P + 0{,}424 \cdot 10^{-6} (h_P - N_P) + TC_P [m^2 s^{-2}] \qquad (7)$$

In formulas (6) and (7) $\overline{g}_P$, is the average gravity between *P* and the geoid, h_P is the ellipsoidal height, N_P is the interpolated geoid undulation, and g_P is the observed gravity, all at the IHRF station at *P*. The factor 0.424 x 10^6 refers to half the vertical gravity gradient in Poincaré-Prey theory, with an average topographic mass density of 2670 kg m³ and TC_P is the topographic correction.

2.2 ZERO-ORDER TERM

The zero-order term includes the difference in the values of the *GM* parameter used by the chosen global geopotential model and the GRS80 reference ellipsoid. Care must be taken to ensure that the first part of equation 8 or 9 has already been taken into account in the chosen geoid or quasi-geoid model. If so, only the second part of the formulas (which is the difference between the values of the potential W_0 adopted by the IHRS and the potential U_0 of the GRS80) should be adopted; otherwise, use the complete formulas. The zero-order term for a quasi-geoid and geoid model is as follows:

$$\zeta_0 = \frac{GM_{GGM} - GM_{GRS80}}{r_P \gamma_Q} - \frac{W_0 - U_0}{\gamma_Q} \quad (8)$$

$$N_0 = \frac{GM_{GGM} - GM_{GRS80}}{r_{P_0} \gamma_{Q_0}} - \frac{W_0 - U_0}{\gamma_{Q_0}} \quad (9)$$

where *GM* is the geocentric gravitational constant of the global geopotential model, , r_P and r_{P_0} are the geocentric radial distances of point *P* (presented in equation (3)), γ_Q is the theoretical gravity at point *Q* on the telluroid, and γ_{Q_0} is the theoretical gravity on the reference ellipsoid (see Figure 1).

Once the ζ_0 has been determined for the quasi-geoid model or the N_0 for the geoid model, the height anomalies or geoid undulations in equations 4, 5, 6, and 7 must be corrected beforehand, as follows: $h_P - (\zeta_P - \zeta_0)$ and $h_P - (N_P - N_0)$, as appropriate.

2.3 TIDE TREATMENTS

It is necessary to note the permanent tide concept used for the input data (the GNSS coordinates and the GGM used in the calculation of the geoid or quasi-geoid model) and then make the necessary corrections to the intermediate results so that the geopotential number is expressed in the zero-tide concept (C_{ZT}) and, consequently, determine the C^{IHRF} in the mean-tide concept. Figure 2 illustrates four different possibilities relating the input data and tide concepts. In the first situation (Figure 2 in pink), if the GGM (C_{20}^{NT}) is in the tide-free concept and the GNSS coordinates are in the mean-tide concept (X^{NT}), the provisional potential value (W_{prov}) is calculated and then the correction (($\Delta\overline{W}^{MGG}$) is applied to obtain the potential at zero-tide (W_{ZT}). In the second situation (Figure 2 in green), the MGG (C_{20}^{ZT}) is in zero-tide, while the GNSS coordinates are in tide-free (X^{NT}). In this case, the correction applied is to correct the GNSS coordinates to the zero-tide system (ΔW^{ITRF}). In the third case (Figure 2 in gray), both the MGG (C_{20}^{NT}) and the coordinates (X^{NT}) are in the tide-free concept and both must be corrected to the zero-tide concept ($\Delta\overline{W}^{MGG} + \Delta W^{ITRF}$). Finally, the last case does not require correction of the intermediate result, since the input data (C_{20}^{ZT} and X^{MT}) are in the zero-tide and mean-tide concepts, respectively. For the particular case of IHRF stations in the SIRGAS region, only the green and gray possibilities can occur since the coordinates are in the ITRF.

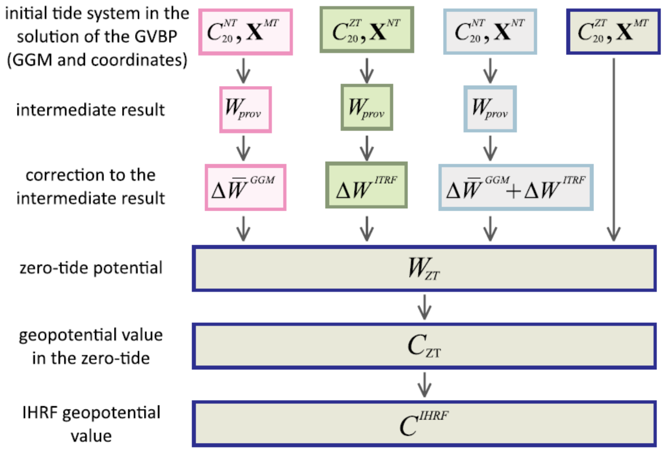

Figure 2 Diagram for determining geopotential values based on the tide concept of GNSS coordinates and the global geopotential model (Modified from Sánchez et al. 2021).

The mathematical formulation regarding corrections is presented below (Sánchez et al. 2021). Equation (10) must be applied if the potential was calculated with a global geopotential model in the tide-free concept. This correction is calculated using the formula:

$$\Delta\overline{W}^{GGM}(\varphi,h) = k_{20}\cdot\left(1-\frac{3h}{a}\right)\cdot(0{,}9722-2{,}8673\cdot sen^2\varphi-0{,}0690\cdot sen^4\varphi)[m^2s^{-2}] \qquad (10)$$

If the coordinates of the point are in the tide-free tide concept (such as the (ITRF of SIRGAS) coordinates), the formula for the correction ΔW^{ITRF} is:

$$\Delta W^{ITRF}(\varphi) \approx \left(-\gamma_0(\varphi)\right)\cdot h_T(\varphi) = -0{,}5901+1{,}7475\cdot sen^2\varphi+0{,}0273\cdot sen^4\varphi[m^2s^{-2}] \qquad (11)$$

where is the normal gravity at the surface of the ellipsoid and $h_T(\varphi)$ the projection of the vector $\Delta\overline{r}$ (see Mäkinen (2021)).

If both corrections (10) and (11) are needed, they can be combined in this case (at *h = 0* and the Love number, k_{20} = 0.30190):

$$\Delta W^{ITRF} + \Delta\overline{W}^{GGM} = -0{,}2966+0{,}8819\cdot sen^2\varphi+0{,}0065\cdot sen^4\varphi[m^2s^{-2}] \qquad (12)$$

Once the corrections have been made, the geopotential in the zero-tide tidal concept takes the following form:

$$W_{ZT} = W_{prov} + \Delta W^{ITRF} + \Delta\overline{W}^{GGM} \qquad (13)$$

The geopotential number in the concept of zero-tide tides is defined as:

$$C_{ZT} = W_0 - W_{ZT} \qquad (14)$$

Finally, the correction of the temporal component generated by the permanent mean tidal potential, calculated at the ellipsoidal height *h = 0* in GRS80, must be applied using the following formula:

$$W_{T0} \approx \overline{W}_T(\varphi, 0) = 0{,}9722 - 2{,}8841 \cdot sen^2\varphi - 0{,}0195 \cdot sen^4\varphi [m^2 s^{-2}] \quad (15)$$

Finally, the IHRF geopotential number is defined and calculated by:

$$C^{IHRF} := C_{ZT} - W_{T0} \quad (16)$$

3. NUMERICAL APPLICATION

To enable interested parties to verify their tools and programs, we provide numerical data from two stations located in Uruguay. The data were kindly provided by the Military Geographic Institute of Uruguay (IGM, 2023a) (IGM, 2024), in a document available to those interested in this WGIII or directly through the IGM. In Table 2, the coordinates (latitude, longitude, and ellipsoidal height) are in the tide-free or Non-Tidal (NT) concept. The height (*H*) refers to the Cabildo datum. The height anomaly (ζ) and geoidal undulation (*N*) were interpolated from the quasi-geoidal model UruQGeoide110 (IGM, 2023b) and the geoidal model UruGeoide110 (IGM, 2023c), respectively. The MGG used to calculate the geoid or quasi-geoid model is based on the zero-tide concept. The gravity value at each station was reduced to the base of the CORS antennas, where the coordinates are referenced. For this numerical calculation, equations 8 and 9 should only be considered in the second term, since the first was already considered in the calculation of the geoid or quasi-geoid. It should also be noted that, for corrections to the geopotential value, only equations 11 and 15 should be considered (second case in Figure 2). For the calculations, it is recommended to use three decimal places for the values of geoidal undulation or height anomaly and for the values of the zero-order term (equations 8 and 9). For the values of theoretical gravity (equation 2) and mean gravity (equation 7), use eight decimal places to represent them. For the calculation of the provisional W_P (equations 4 or 6) and for the corrections (equations 10 to 15), use three decimal places in the results. The final values of the C_{IHRF} (equation 16) shall be rounded to two decimal places. This ensures that the results are compatible when compared to the results calculated by the GT-III.

Table 2 Numerical values for calculating potential values

Station	Latitude	Longitude	h (m)	H (m)	ζ (m)	N (m)	g (observed) (Gal)
UYPT	-32.80055949	-56.50981698	91.118	74.299	16.059	16.060	9.79557947
UYTA	-31.68306443	-55.93753385	186.981	171.523	14.680	14.678	9.79414841

For comparison and validation purposes, interested parties may share their results with WGIII, including results from intermediate steps for better verification. In any case, we share the results of the experiment (Table 3 considering the quasi-geoid model UruQGeoide110 and Table 4 for the geoid model UruGeoide110).

Table 3 Results for stations UYPT and UYTA using data from UruQGeoide110

Quantity	Equation	UYPT	UYTA
γ_0	2	9.79549779	9.79458678
ζ_0	Second term of equation 8	-0.761	-0.761
$\bar{\gamma}_{QQ_0}$	5	9.79538314	9.79432205
W_P	4	62636125.642	62635173.282
$\Delta W^{ITRF}(\varphi)$	11	-0.075	-0.106
W_{ZT}	13	62636125.567	62635173.176
C_{ZT}	14	727.833	1680.224
W_{T0}	15	0.124	0.175
C^{IHRF}	16	727.71	1680.05

Table 4 Results for stations UYPT and UYTA using data from UruGeoide110

Quantity	Equation	UYPT	UYTA
γ_0	2	9.79549779	9.79458678
N_0	Second term of equation 9	-0.761	-0.761
$\bar{g}_P$	7	9.79561371	9.79422567
W_P	6	62636125.635	62635173.279
$\Delta W^{ITRF}(\varphi)$	11	-0.075	-0.106
W_{ZT}	13	62636125.560	62635173.173
C_{ZT}	14	727.840	1680.227
W_{T0}	15	0.124	0.175
C^{IHRF}	16	727.72	1680.05

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