



EPUSP

Escola Politécnica da Universidade
de São Paulo



UNIVERSIDADE FEDERAL
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EFFORTS FOR THE REGIONAL IHRF COMPUTATION IN THE SÃO PAULO STATE

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SUMMARY

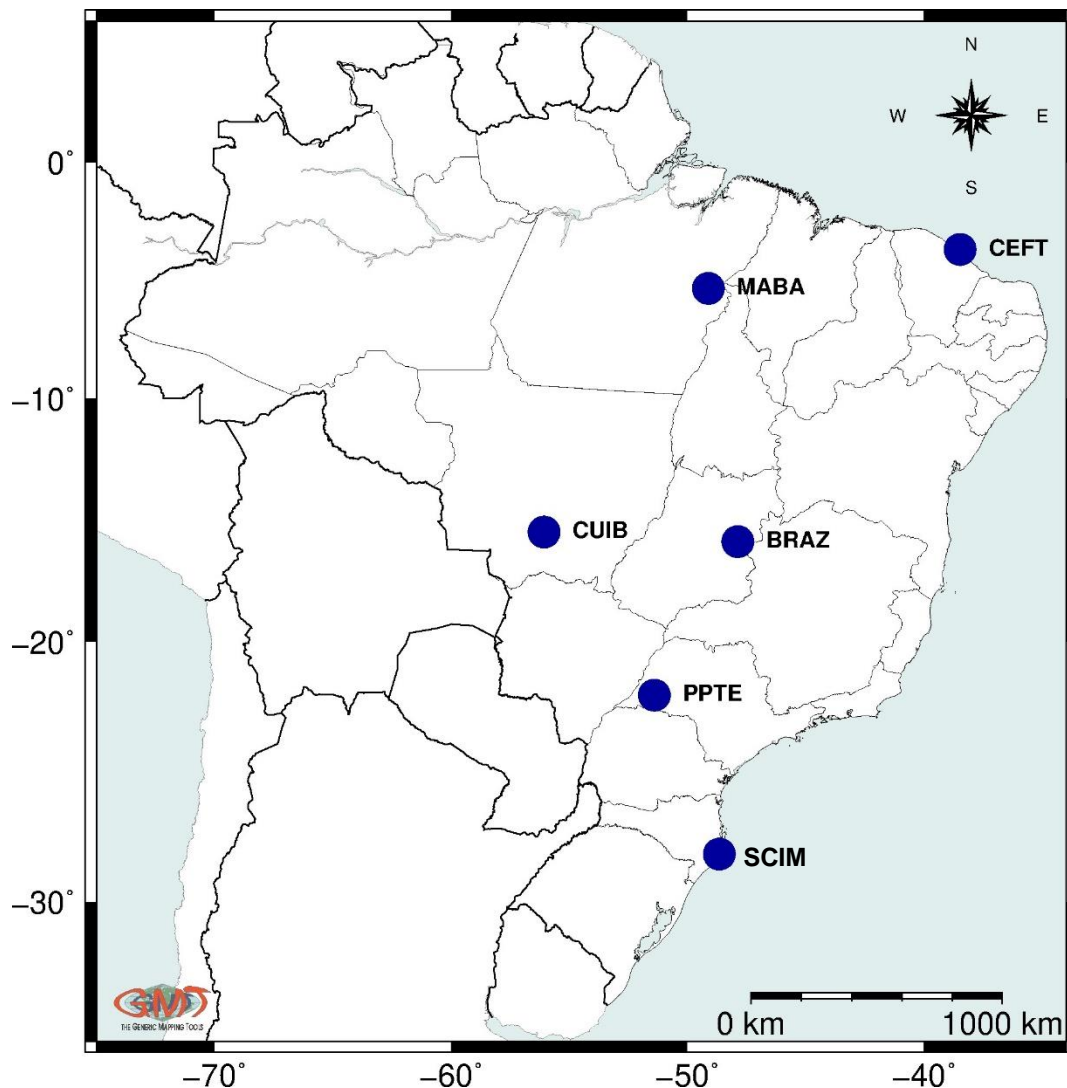
- 1) IHRF points in Brazil
- 2) Hotine Integral
- 3) IHRF in the state of São Paulo
- 4) The modified Hotine kernel
- 5) Results of IHRF in São Paulo
- 6) Origins of height system in Brazil
- 7) Final considerations and conclusion

IHRF in Brazil

Six stations were selected in Brasil by IBGE for IHRF determination: Brasilia (BRAZ), Cuiabá (CUIB), Imbituba (SCIM), Marabá (MABA), Presidente Prudente (PPTE).

❖ The gravity coverage of these stations are still dependent of complementary surveys. IBGE is doing efforts for this purpose and the present attention is addressed to Marabá.

Figure 1: IHRF stations distribution in Brazil



Hotine Integral

The gravity disturbance used in the Hotine integral is defined as follow:

$$\delta g(P) = g_P - \gamma_P$$

The normal gravity is upward continued from the ellipsoid to P.

Hotine integral

The Hotine integral for the disturbing potential T reads:

$$T(P) = \frac{R}{4\pi} \iint \delta g H(\psi) d\sigma$$

where δg is the gravity disturbance in the observed points P .

Numerical integration has been used with irregular gravity data distribution.

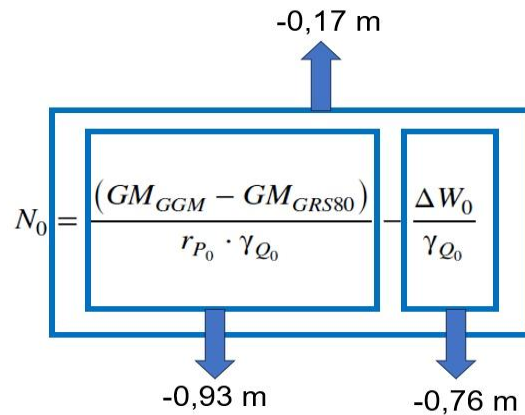
Height system

In 2015, the General Assembly of the *International Union of Geodesy and Geophysics* (IUGG), the International Association of Geodesy (IAG) recommended to adopt a reference value for the gravity potential in the Surface of origin, with the objective to establish the IHRs/F (International Height Reference System/Frame).

$$W_0 = (62.636.853,4 \text{ m}^2\text{s}^{-2})$$

The challenge now is to establish points with the gravity potential determined in order to have the difference with respect to W_0 ; in this way to compute the normal height.

Zero order terms involved



$$T_o = 1/r_p (GM_{GGM} - GM_{GRS80}) = 1/r_p (3,986004415 \times 10^{14} \text{ m}^3\text{s}^{-2}$$

$$- 3,98600 \times 10^{14} \text{ m}^3 \text{ s}^{-2})$$

$$\Delta W_o = W_o - U_o = (62.636.853,4 \text{ m}^2 \text{ s}^{-2} - 62.636.860,85 \text{ m}^2 \text{ s}^{-2})$$

IHRF in the state of São Paulo

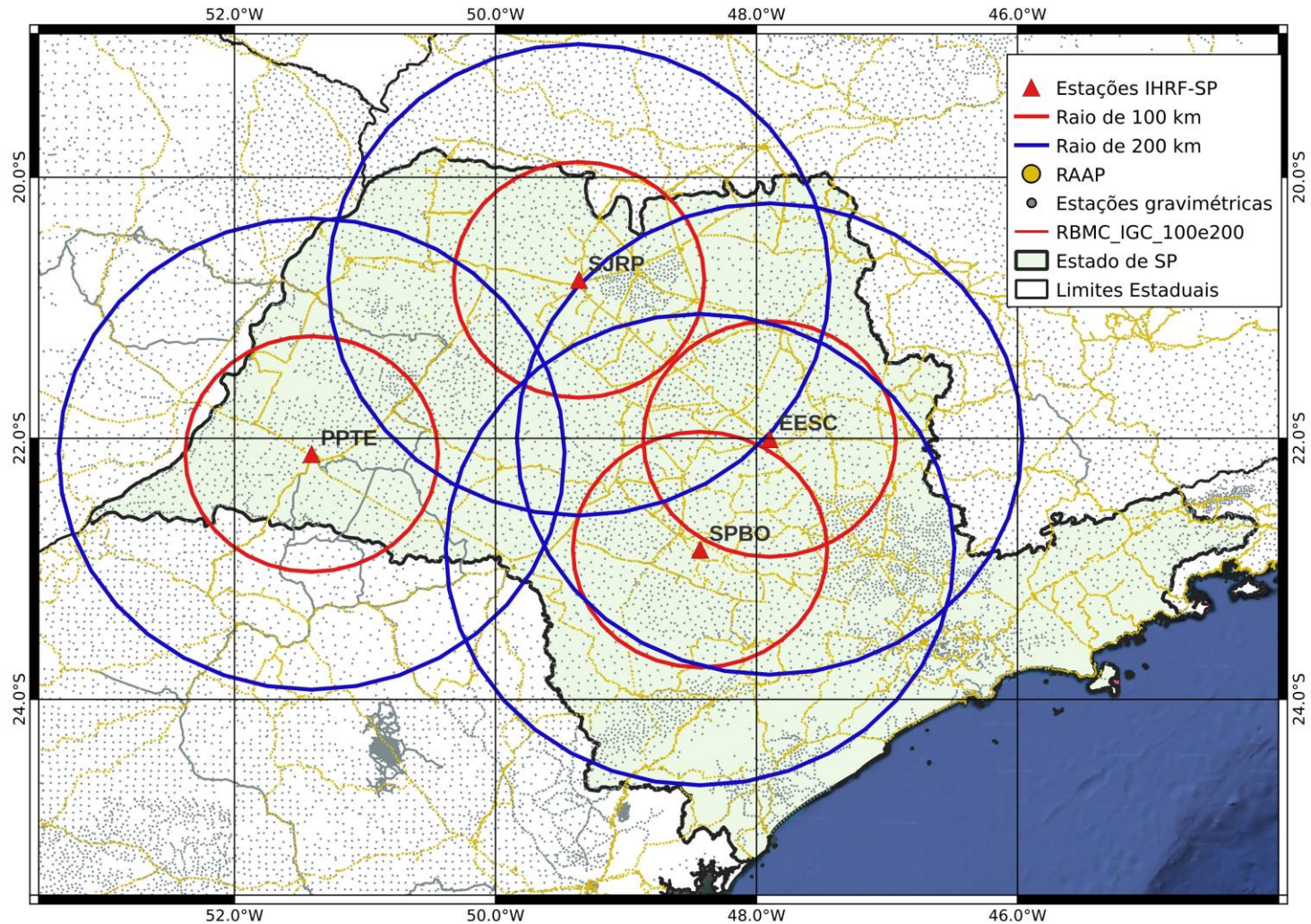
In the state four stations have been selected for IHRF determination: Presidente Prudente (PPTE), São Carlos (EESC), São José do Rio Preto (SJRP) y Botucatu (SPBO).

g

❖ The state has a good and quite homogeneous gravity coverage, due to the so called Projeto Temático (FAPESP 2006/04008-2) and complementary efforts.

❖ As a consequence it has been possible to select four points with excellent possibilities for the determination of the disturbing potential.

IHRF stations in São Paulo and gravity data distribution



Softwares for Hotine integral

Two softwares have been established by CENEGEO: ***hotmod.f*** for Hotine kernel modification. The objective is to make compatible the convergence of the kernel with the distant Ψ_0 from the computation point, as a function of the geopotential model degree and order used.

The second software: ***hotine-ptg-WP-tk.f*** It performs the numerical integration of Hotine integral formula, using gravity point, eventually in a irregular distribution.

The figures show the normal kernel, the Wong-Gore and the present modifications

Modified Hotine (Neumann-Koch) integral

$$T_l(\theta, \lambda) = \frac{R}{4\pi} \left[\int_{\psi=0}^{\psi_0} \int_{\psi=0}^{360} \delta g(\alpha, \psi) \delta H_l^m \operatorname{sen}(\psi) \partial\psi \partial\alpha \right]$$

$$H(\psi) = \operatorname{csc}(\psi / 2) - \ln\left(1 + \operatorname{csc}\left(\frac{\psi}{2}\right)\right) = \sum_{n=0}^{\infty} \frac{2n+1}{n+1} P_n(\cos(\psi))$$

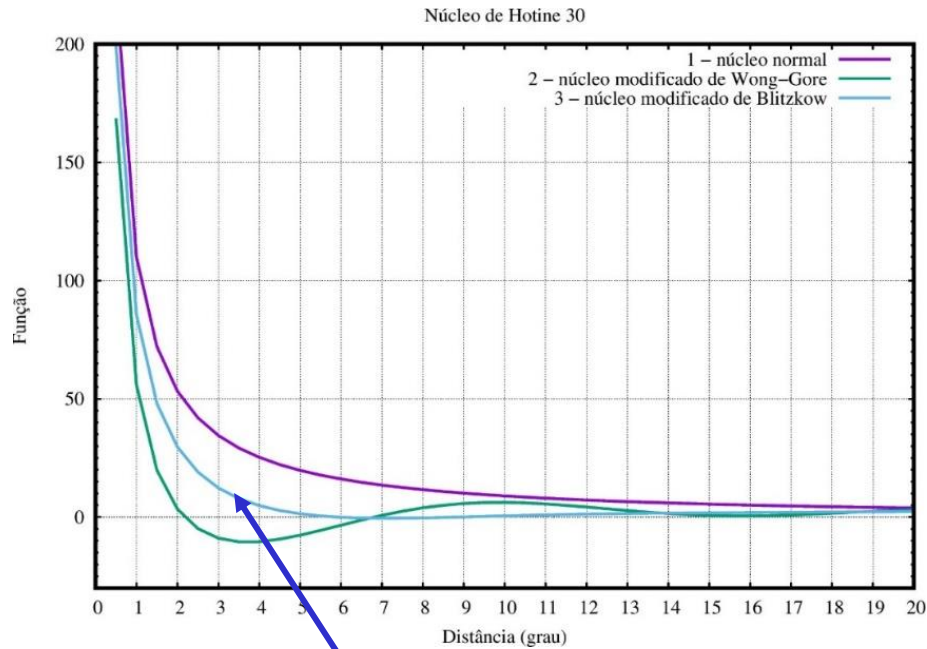
$$\delta H_l^m(\Psi) = \delta H_l(\Psi) - \overline{\delta H_l}(\Psi)$$

$$\delta H_l(\Psi) = H(\Psi) - H_l(\Psi)$$

$$H_l(\Psi) = \sum_{n=2}^l \frac{2n+1}{n+1} P_n(\Psi)$$

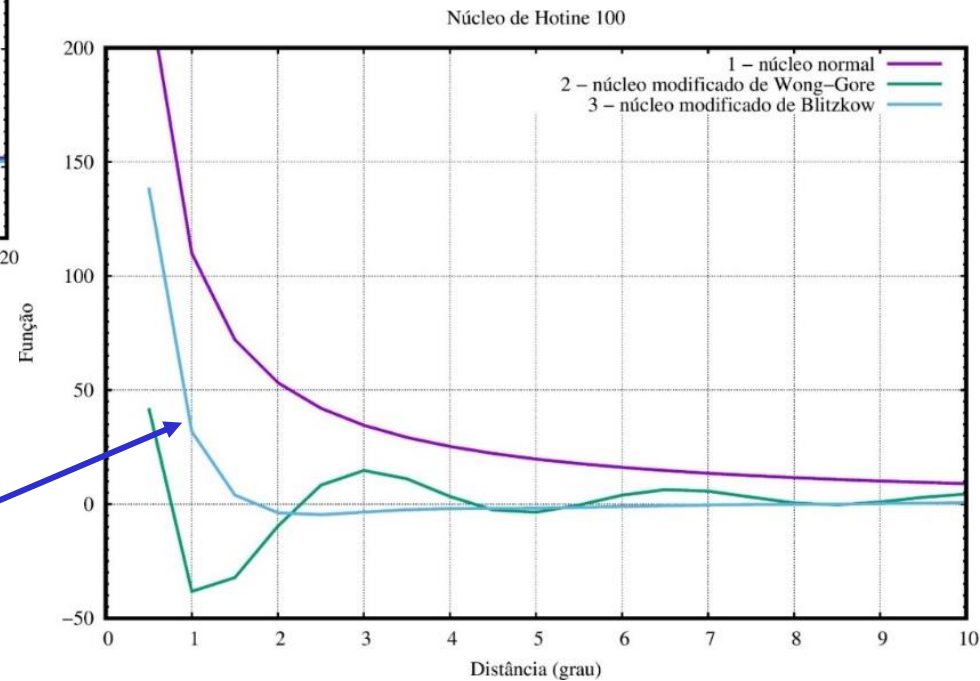
$$\overline{\delta H_l}(\Psi) = \sum_{i=0}^l \frac{2i+1}{2} t_i P_i(\cos \Psi)$$

Modified Hotine Hotine



Degree and order 30

Modified kernel for degree
100 and 30



Degree and order 100

São Paulo IHRF results.

Estação	Normal height-Hotine (m)	Normal height-G100 (m)	Normal height-G360 (m)	Geodetic height (m)	Difference Hotine Q100 (m)	Difference Hotine Q360 (m)	
EESC	830,053	830,642	830,669	824,587	-0,59	-0,62	
SJRP	542,212	542,677	542,663	535,909	-0,47	-0,45	
SPBO	807,995	808,431	808,476	803,122	-0,44	-0,48	
PSTE	435,854	435,868	435,864	431,049	-0,01	-0,01	

The results are in zero tide concept. It is needed to convert to mean-tide to be consistent with IHRS conventions. The geoid model was converted to normal height using the procedure described at Laura et al., 2021: Strategy for the realisation of the International Height Reference System (IHRS).

São Paulo IHRF results.

Heights from Geoid and Hotine values are consistent to about 50 cm.

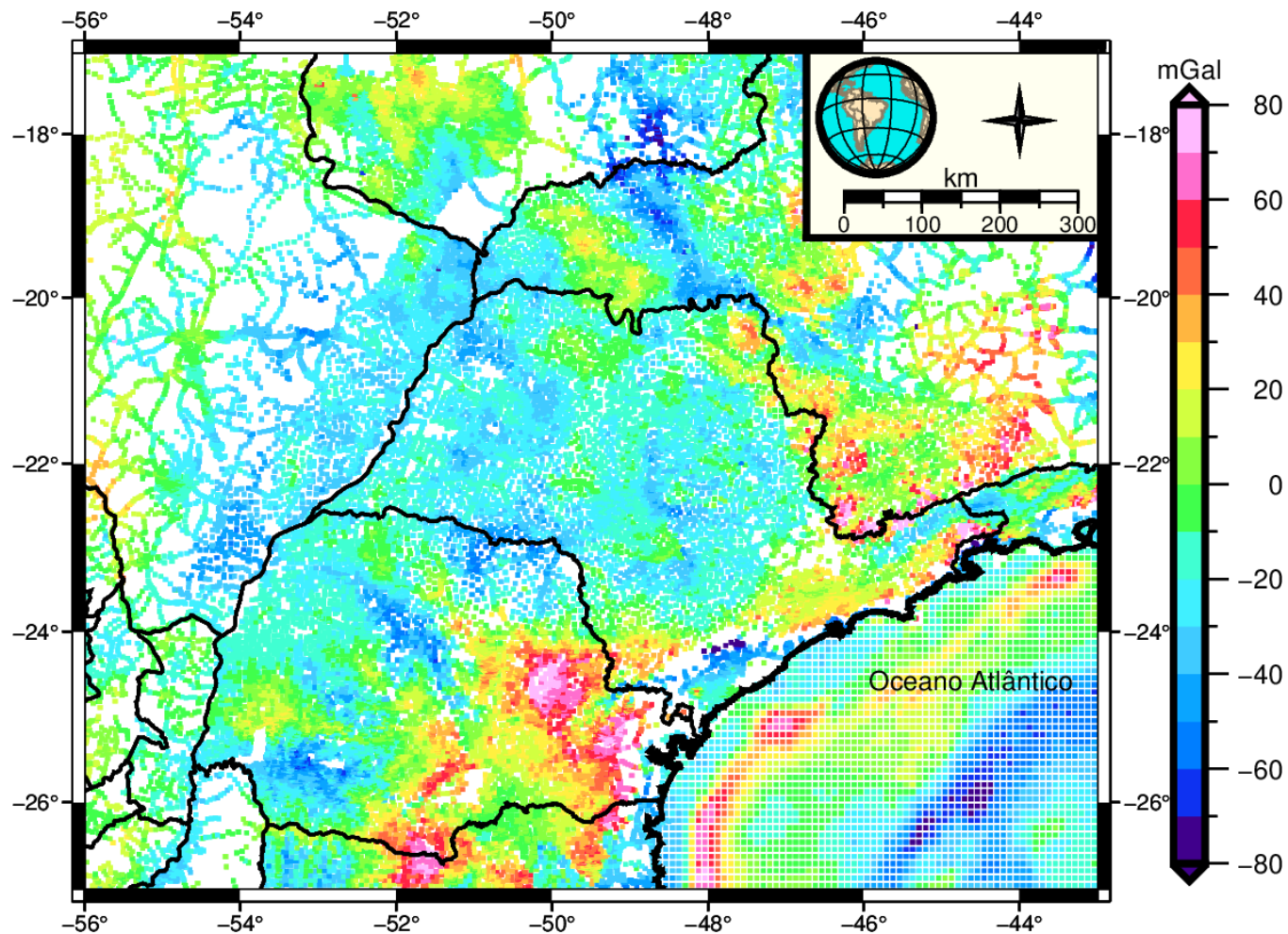
It is expected some difference due to the fact that the geoid is derived from 5' mean anomaly and Hotine integral is accomplished point-by-point. But, the main problem is that, the value of T at the physical surface needs a "free air gradient correction", out of the Bouguer plate correction.

At PPTE there is a value of H_N as a result of the IBGE levelling.

IBGE H_N : **436,367 m**. Difference: IBGE - Hotine = **0,513 m**

The IBGE value of H_N is in the concept of mean tide. When converted the Hotine height to mean tide, the difference reduces to **0,429 m**.

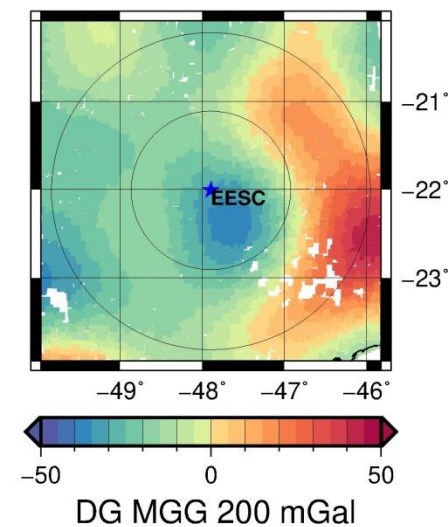
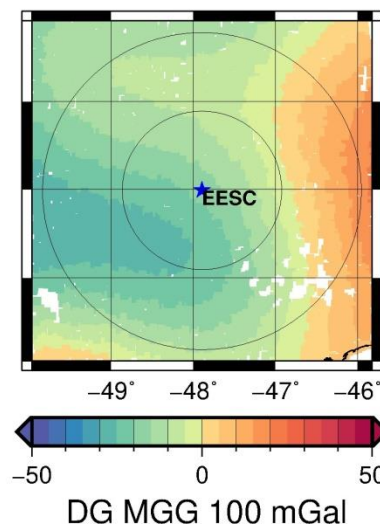
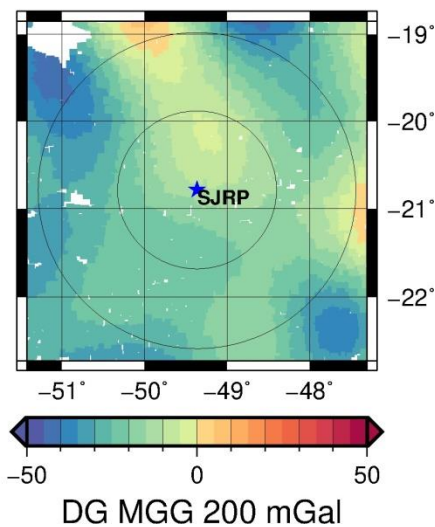
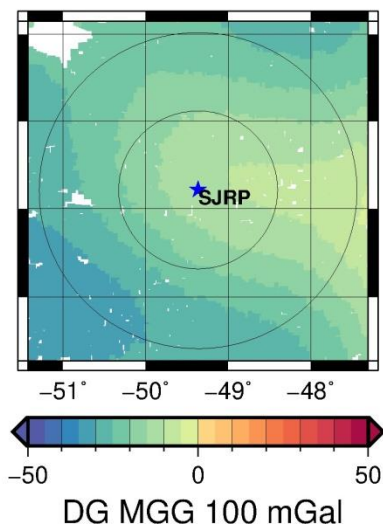
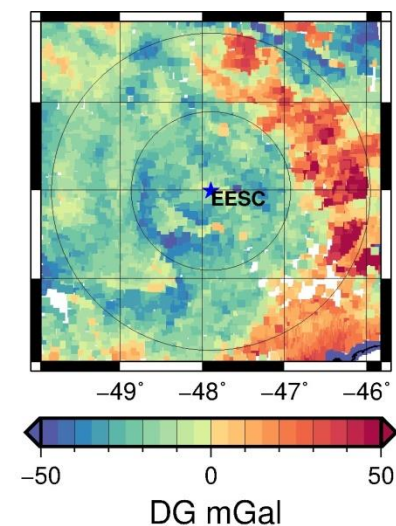
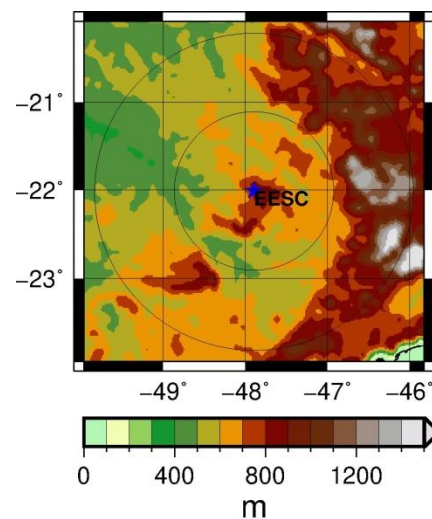
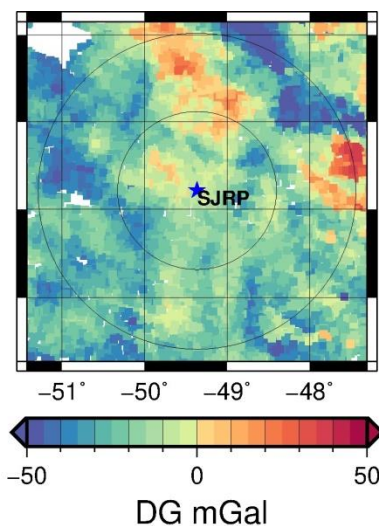
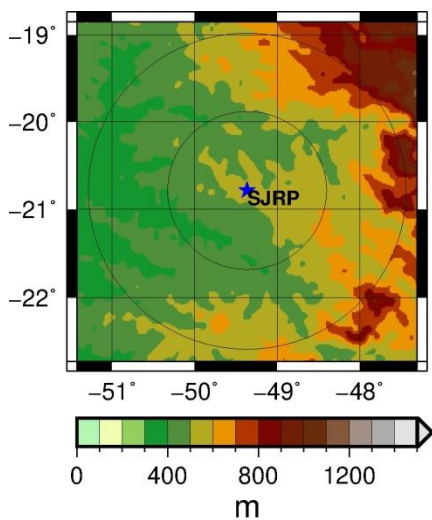
Gravity disturbances in the São Paulo state area



Topography, gravity distances from point data and from GGM in the IHRF stations.

SJRP

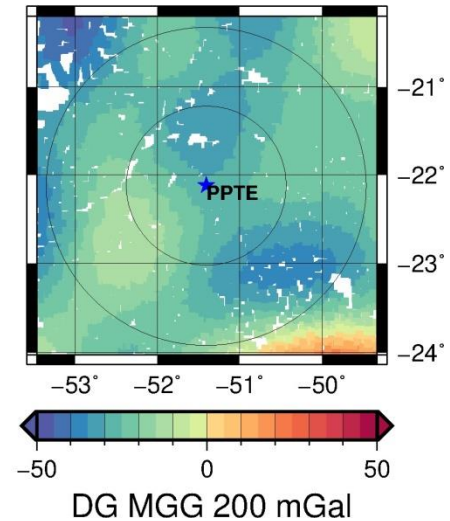
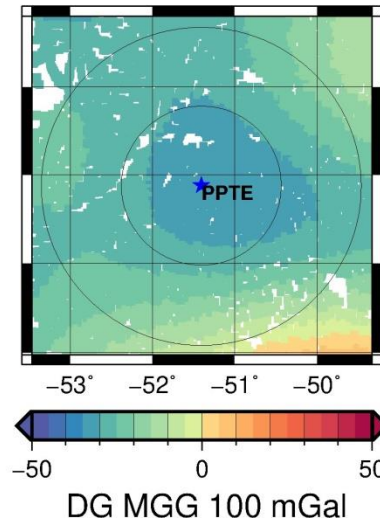
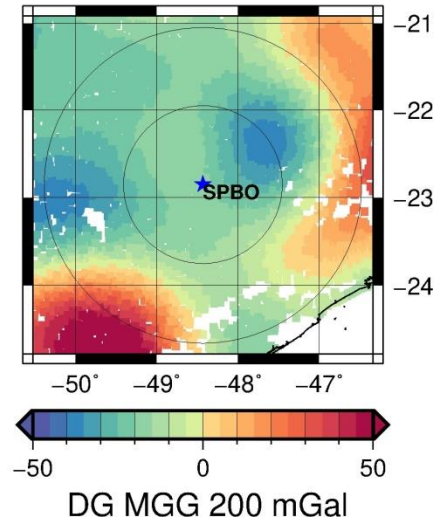
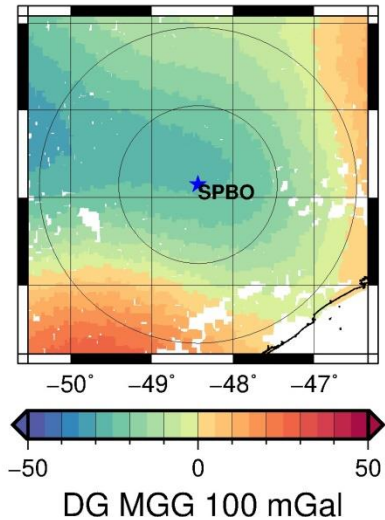
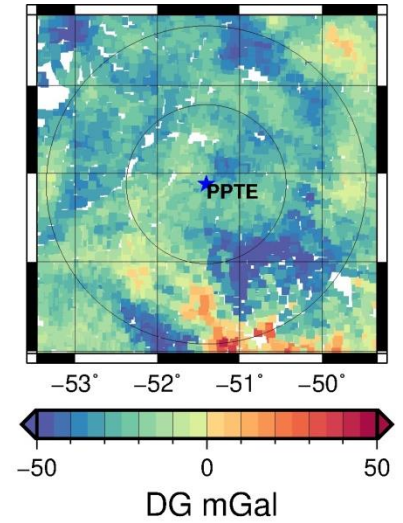
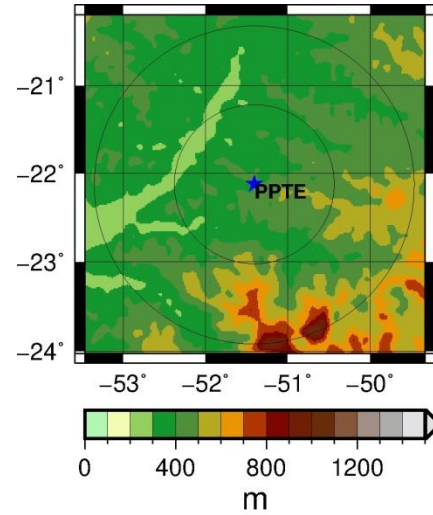
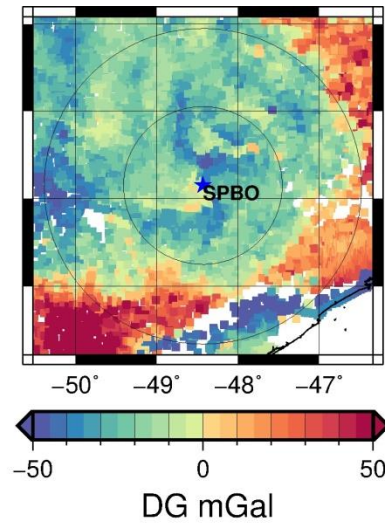
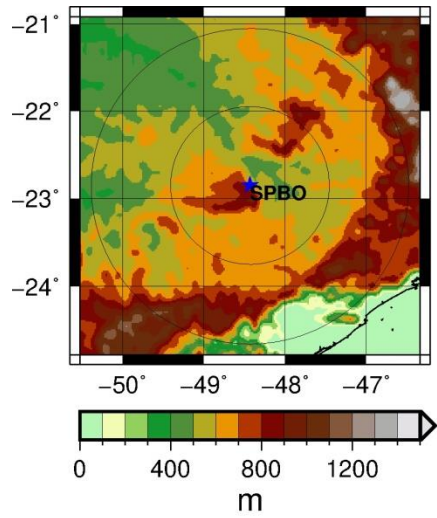
EESC



Topography, gravity distances from point data and from GGM in the IHRF stations.

SPBO

PSTE



Height system origins in Brazil

An adjustment of the levelling network was undertaken in 2018 by IBGE. Two origin was used: Imbituba and Santana. The adjustment processing was carried out in terms of geopotential numbers. As reference values in the origin for the adjustment were RN 4X and RN 9329T, in Imbituba and Santana respectively:

$$4X = 84,565 \text{ m}^2 / \text{s}^2$$

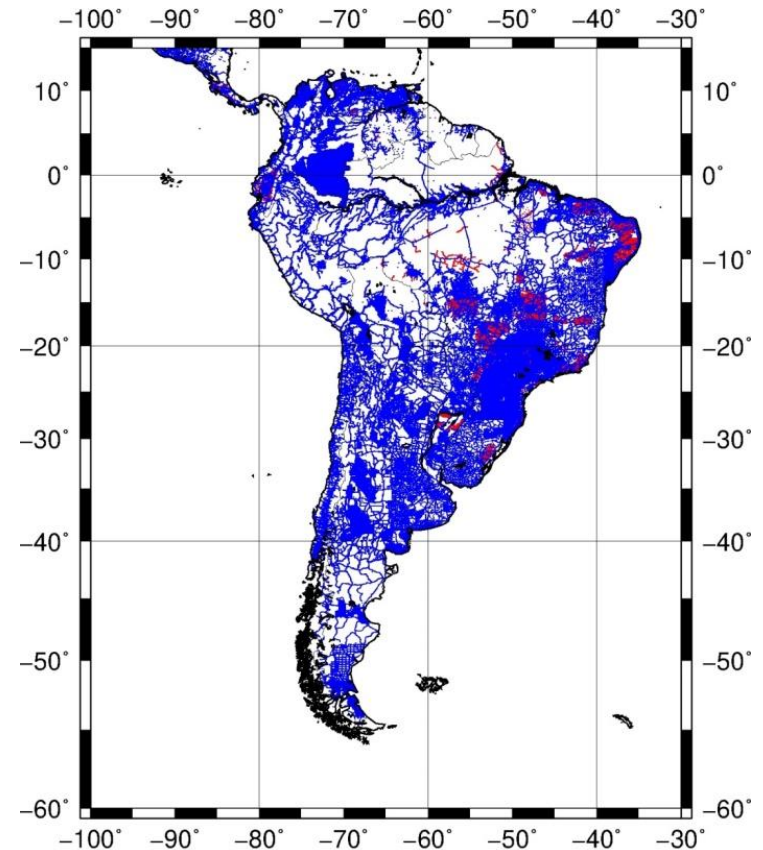
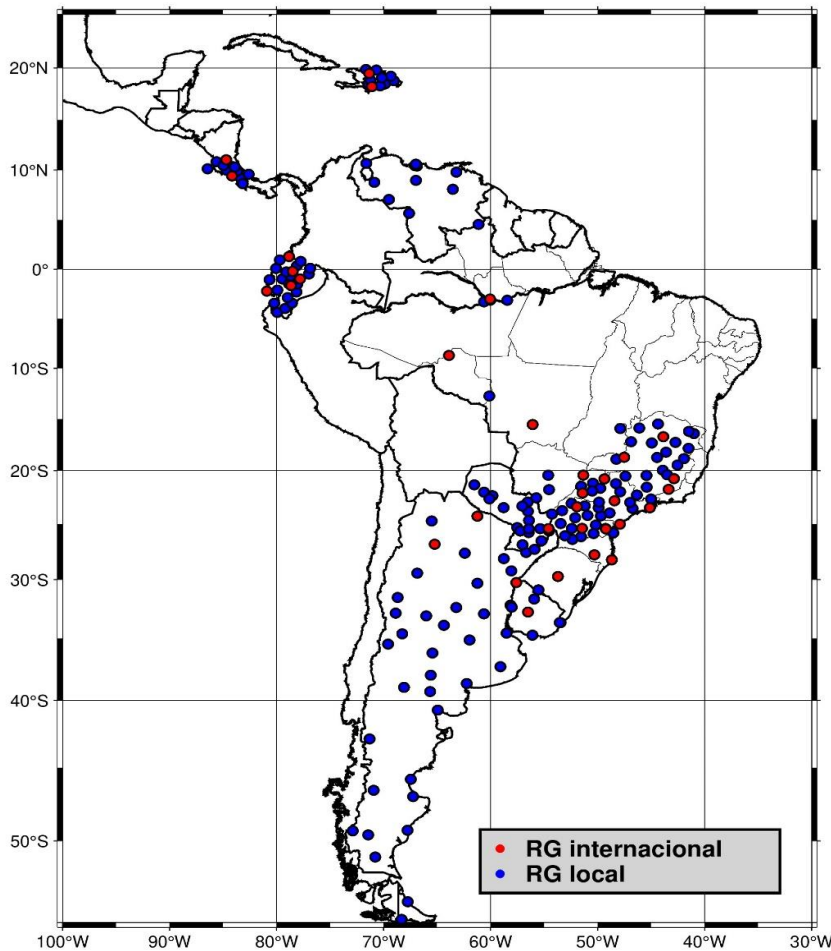
$$9329T = 61,401 \text{ m}^2 / \text{s}^2$$

The values were derived by simply transforming the result of the levelling height, from the tide-gauge, to geopotential numbers. It is a preliminary approximation to the normal height.



RN-4X Imbituba

To finalize, nowadays it is fundamental to associate the IHRF, the geoid model and the Reference Gravity. At CENEGEO we are addressing our attention to these important issues.



Surveys gravity
▲ New points ▲ gravity up to 2015



Final considerations and conclusions

- 1) In terms of Brazil IBGE is working hard to improve the gravity data distribution, in particular, around IHRF points;
- 2) São Paulo state has an optimistic coverage of gravity data for IHRF determination.
- 3) The numerical integration was undertaken in a point-by-point procedure.
- 4) PPTTE is a good example that shows the importance to use the GGM to degree 100 and extend the integral up to 1.8° ($180/100$) degree, due to the existence of important residual gravity anomalies in the area between 100 and 200 km.
- 5) The coordinates for the computation were always derived from RBMC in the points.

Final considerations and conclusions

- 6) Hotine integral with a convenient modification of the kernel results in an option for normal height determination. PPTTE is the only point in São Paulo with a height derived from levelling, important for comparison.
- 7) It is important to connect the leveling network of Brazil to the three other IHRF points in São Paulo.
- 8) The transformation from geoidal Height referred to W_0 at points with geodetic coordinates has to be improved with a "free air gradient correction" for the disturbing potential.



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Acknowledgments



Gracias
Thank you

