Global vertical datum unification based on the combination of the fixed gravimetric and the scalar free geodetic boundary value problems

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SIRGAS contribution to the IAG-ICP1.2: Vertical Reference Frames

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Motivation

World Height System (WHS)

(IAG-ICP1.2: Vertical Reference Frames, Ihde et al. 2007)

Consistent modelling of geometric and physical parameters, i.e.
$h = H^N + \zeta$ ($\approx H + N$) in a global frame with high accuracy (**> 10^{-9}**)

Geometrical Component

Coordinates:

h (t), dh/dt

Definition:

ITRS + Level ellipsoid ($h_0 = 0$)

a. (**a**, J2, ω, GM) **or**
b. (**W_0**, J2, ω, GM)

Realization:

1. Related to the **ITRS** (ITRF)
2. Conventional ellipsoid

Conventions:

IERS Conventions

Ellipsoid constants, W_0, U_0 values, reference tide system have to be aligned to the physical conventions!

Physical Component

Coord.: Potential differences

$-\Delta W_P(t) = W_0(t) - W_P(t)$; $d\Delta W_0/dt$

Definition:

W_0= *const.* (as a convention)

Realization:

1. Selection of a global **W_0** value
2. Determination of the local **$W_{0,j}$** values
3. Connection of **$W_{0,j}$** with **W_0**
4. Geometrical representation of **W_0** and **$W_{0,j}$** (i.e. geoid comp.)
5. Potential differences into physical heights (**H** or **H^N**)

Zero tide system

Considerations on W_0, $W_{0,j}$

- ☑ The reference level (W_0, $W_{0,j}$) for potential differences can **arbitrarily be appointed**, but it is preferred that this level refers to the mean sea level and it shall be derived from **actual observations** of the Earth's gravity field and of the sea surface (Gauss/Listing geoid definition);
- ☑ The **direct determination** of absolute potential values (W_0, $W_{0,j}$) from observational data **is not possible**, adequate **constraints** are required;
- ☑ These constraints (mainly the vanishing of the gravitational potential V at infinity) are **only reliable** in the frame of the **Geodetic Boundary Value Problem** (GBVP); hence, the determination of suitable W_0 or $W_{0,,j}$ values is exclusively feasible by solving the GBVP;
- ☑ This procedure reduces the 'arbitrariness' of the reference level; the **obtained W values** will be in **agreement** with the **geodetic observations** included for solving the GBVP.

Determination of W_0 and $W_{0,j}$

Ocean areas	Land areas

Fixed gravimetric GBVP

- ☑ Estimation of the potential of the **level surface** that best approximates the **mean sea surface**
- ☑ Geometry of the boundary surface (mean sea surface) is known from **satellite altimetry**
- ☑ This value is appointed as the global reference level W_0

Scalar-free GBVP (Molodensky App)

- ☑ Since the observational data included in the boundary conditions **refer to different vertical datums**, we obtain as many $W_{0,j}$ values as existing height systems **j**.
- ☑ GVBP shall **homogeneously** be solved in all datum zones (**j = 1 … J**); i.e, gravity anomalies at ground level, the same GGM, the same reference ellipsoid, etc.

Relationships $W_0 - W_{0,j}$, $W_{0,j} - W_{0,j+1}$ through vertical datum unification strategies

W_0 ($W_{0,j}$) in the GBVP frame

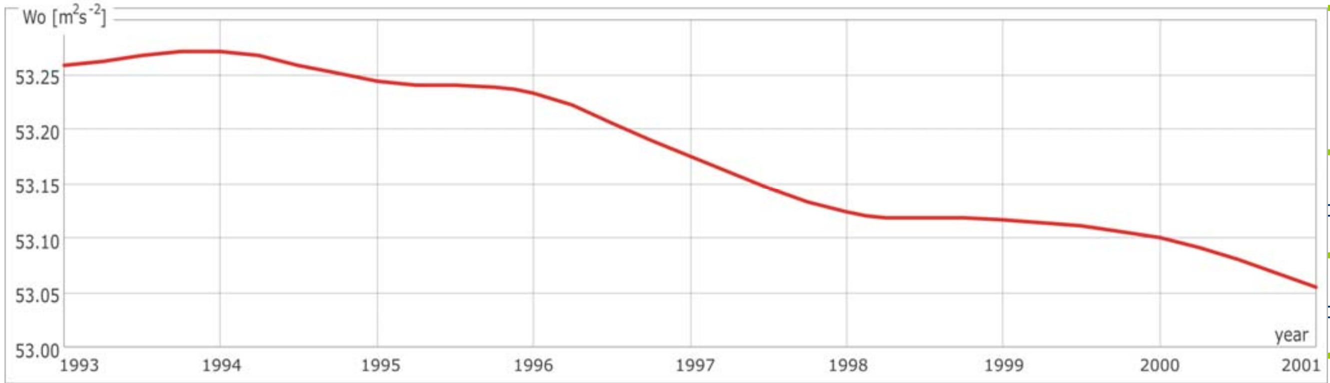
	Ocean areas	Land areas
Formulation	$\nabla^2 T = 0$ *outside boundary surface* ; $T = W - U$	
	$-\frac{\partial T}{\partial r} = \delta g$	$-\frac{\partial T}{\partial r} - \frac{2}{r} T = g_j - \frac{2}{r}\delta W_j$ $\delta W_j = W_{0,j} - U_0 = W_0 - W_{0,j}$
Constraints	$T = 0 \; at \; \infty \, ; \int_{sea} T \, d\sigma = k_i \, ; \int_{land} T \, d\sigma = k_j \, ; \int k_i + \int k_j = 0 \Rightarrow T_{00} \equiv \int T \, d\sigma = 0$	
Solution	$T = \frac{\Delta GM}{R} + \frac{R}{4\pi} \iint_\sigma B_j \, S(\psi) d\sigma + \sum_{n=1}^{\infty} \frac{R}{4\pi} \iint_\sigma G_n \, S(\psi) d\sigma$	
	$j = 1$; $B_1 = \delta g$ (gravity disturbances)	$j = 1 \ldots J$; $B_j = g_j - \frac{2}{r}\delta W_j$ $g_1 = \Delta g$; $g_2 = \Delta C$; etc.
	$S(\psi) = S'(\psi) = \frac{1}{\sin(\psi/2)} - \ln\left(1 + \frac{1}{\sin(\psi/2)}\right)$	$S(\psi) = \frac{1}{\sin(\psi/2)} - 6\sin\frac{\psi}{2} + 1 - 5\cos\psi \ldots$
Results	$W_P = U_0 - \gamma_P h_P + T_P$ $W_0 = \int \frac{W_P}{\gamma_P^2} d\sigma \Big/ \int \frac{1}{\gamma_P^2} d\sigma$	$\zeta_j = \frac{T + \delta W_j}{\gamma}$

Numerical results: W_0 value

Solution of the fixed gravimetric GBVP taken as input data:

Geometry of the mean sea surface: **CLS01** model (Hernandez, Schaeffer 2001),
DGFI annual models derived from T/P

Gravity disturbances from GGM: **EGM2008** model (Pavlis et al. 2008) and
EIGEN-GL04S (GRGS/GFZ 2006)



	… 150)	W_0 [$m^2 s^{-2}$] (EIGEN-GL04S, n = 150)
	… 0,64	62 636 8**54,38** ± 0,64
	… 0,62	**53,83** ± 0,63
60° N…60° S	**53,18** ± 0,55	**53,11** ± 0,56

Other W_0 computations:

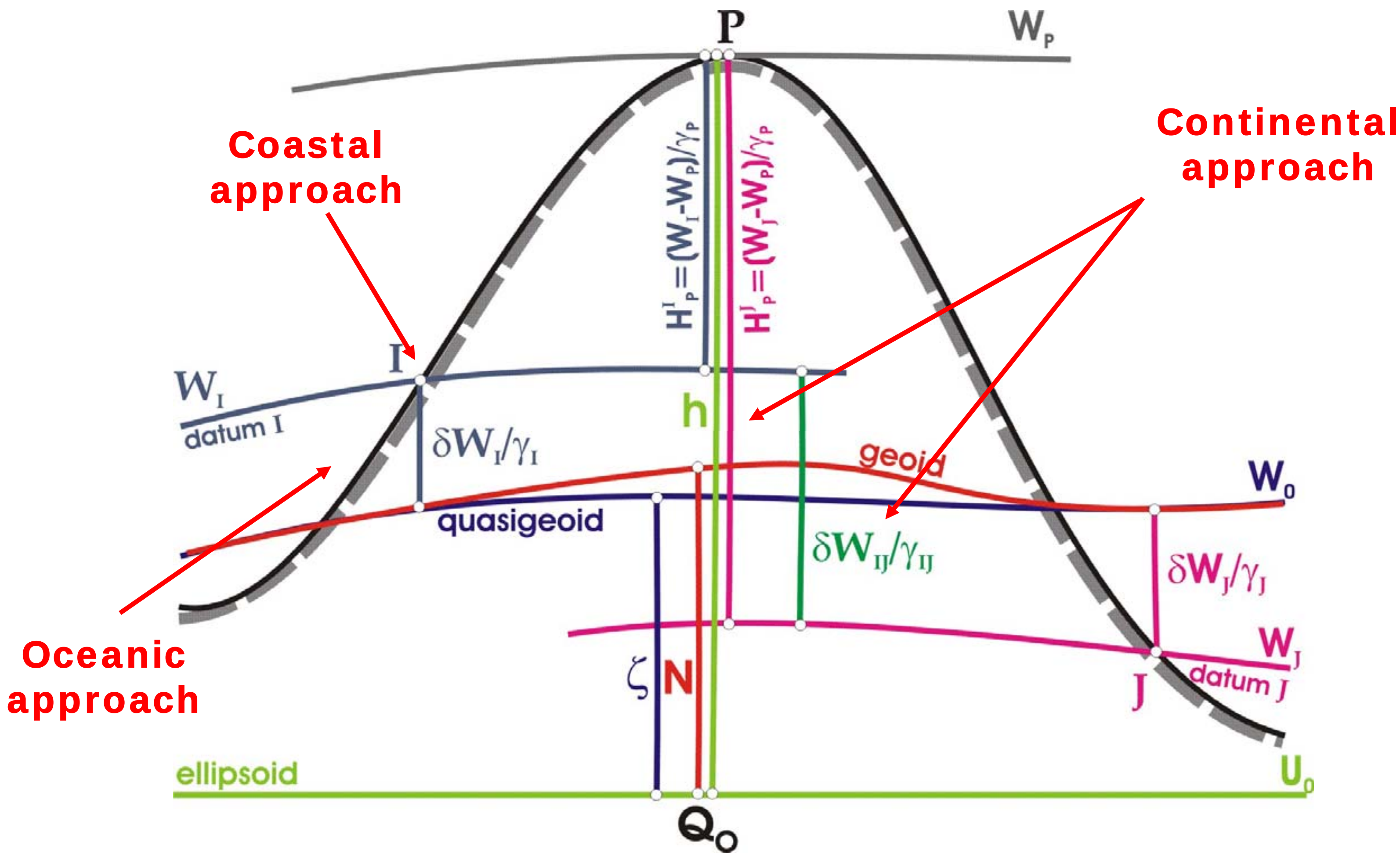
Best fitting ellipsoid:

U_0 = 62 636 8**60,850** m^2s^{-2} (GRS80)
8**56,88** (Rapp, 1995)

Mean potential value from $W = \frac{GM}{r}\left[1+\sum_{n=1}^{\infty}\sum_{m=0}^{n}\left[C_{nm}\cos m\lambda ...\right]\right]$

W_0 = 62 636 8**57,5** (Nesvorny and Sima 1994)
8**56,5** (Ries 1995)
8**56,0** (Bursa et al. 2002)
8**54,7** (Bursa el al. 2006)
8**53,4** (Sánchez 2005)

Vertical datum unification

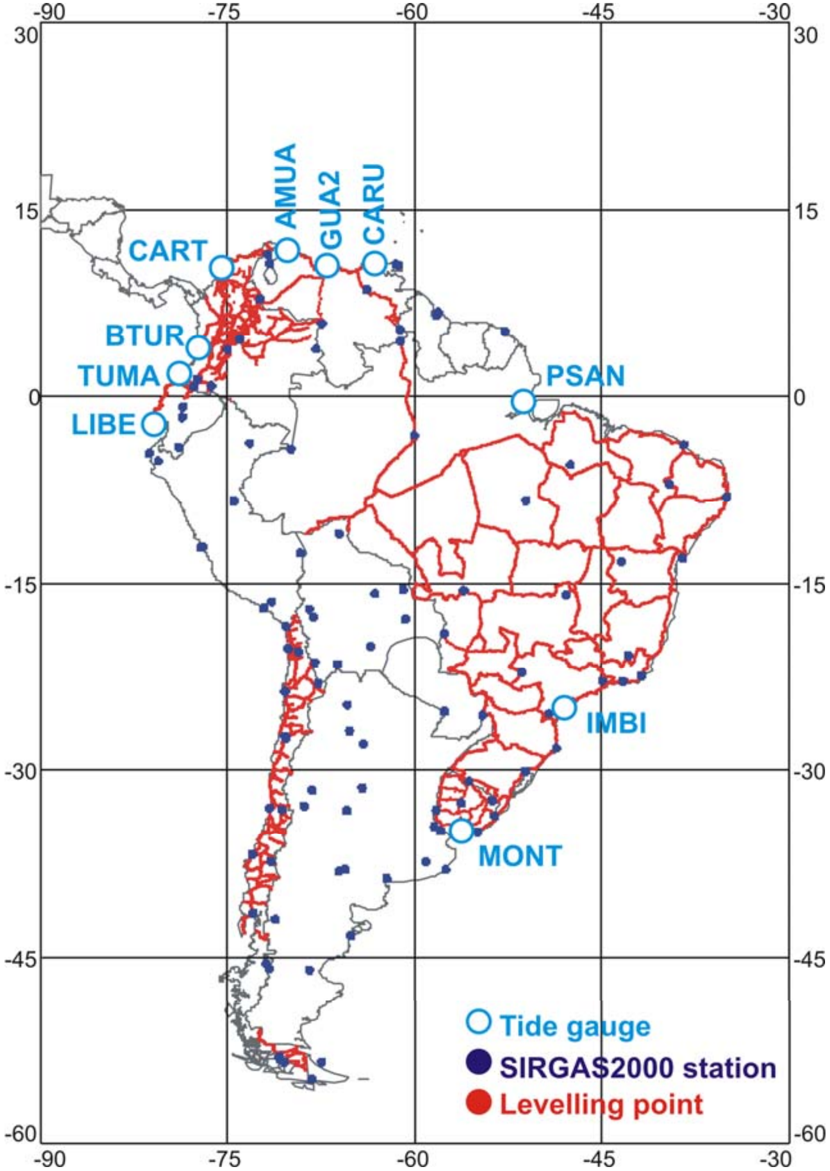


Constraint for the empirical determination of the δW_j terms: $\gamma_P h_P - \left(W_0^j - W_P^j\right) - T_P^j - 2\delta W_j = 0$

Observation equations for Vertical datum unification

Oceanic approach (SSTop around tide gauges) Data: Satellite altimetry and satellite-only GGM, SSTop at coast lines by including also tide gauge records.	$T_P^j - T_0 = \delta W^j$
Coastal approach (reference tide gauges) Data: GPS positioning at tide gauges, spirit levelling with gravity corrections, terrestrial gravity data and satellite-only GGM.	$\frac{1}{2}T_P^j - \frac{1}{2}h_P\gamma_P = \delta W^j$
Continental approach (geometric reference stations) Data: GPS positioning at reference stations (including border points), spirit levelling with gravity corrections, terrestrial gravity data and satellite-only GGM.	$\frac{1}{2}\left(W_0^j - W_P^j + T_P^j\right) - \frac{1}{2}h_P\gamma_P = \delta W^j$ $\frac{1}{2}\left(W_0^j - W_P^j + T_P^j\right) - \frac{1}{2}\left(W_0^{j+1} - W_P^{j+1} + T_P^{j+1}\right) = \delta W^{j+1} - \delta W^j$

Numerical results: SIRGAS example



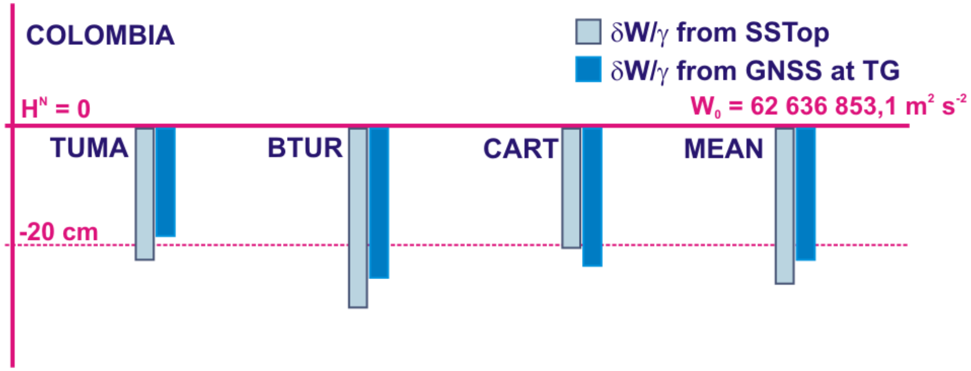
Input data:

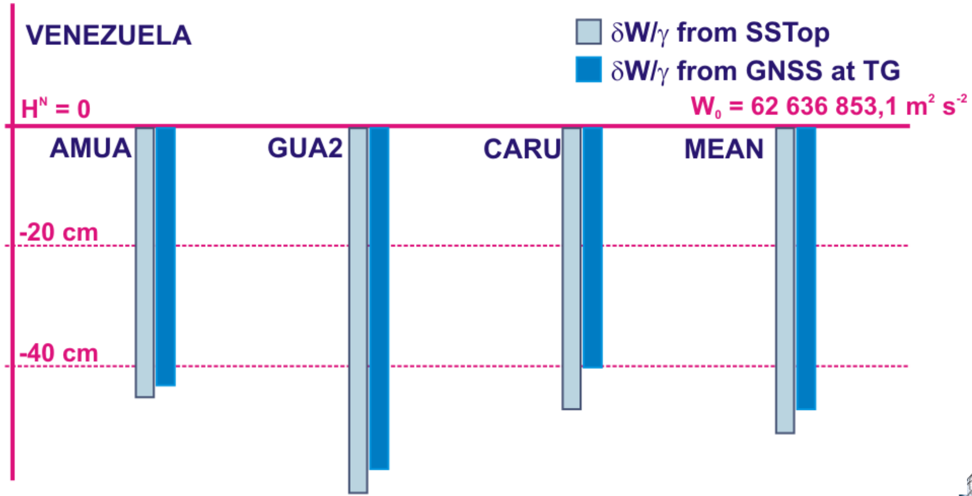
Local quasigeoid models

GNSS positioning, mean sea surface heights,

Geopotential numbers from levelling

H^N, h, ζ, SSTop at epoch 2000.0, zero tide system





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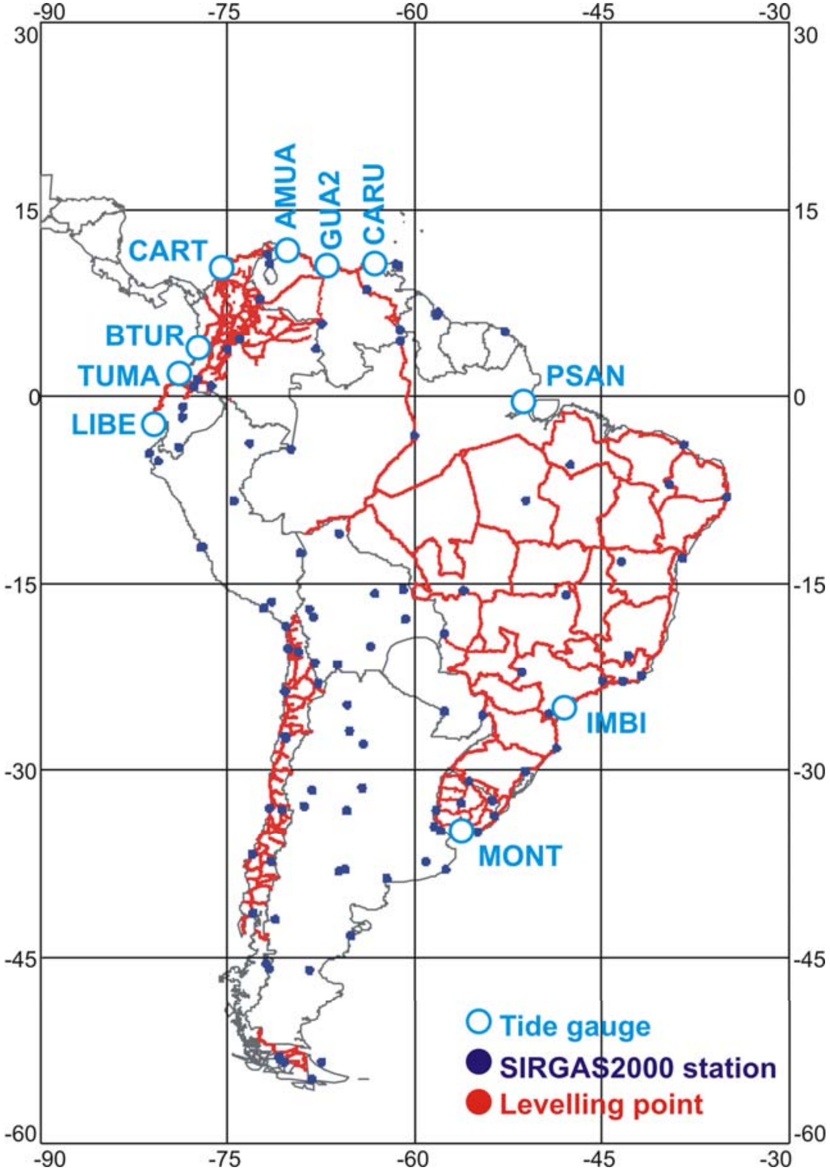

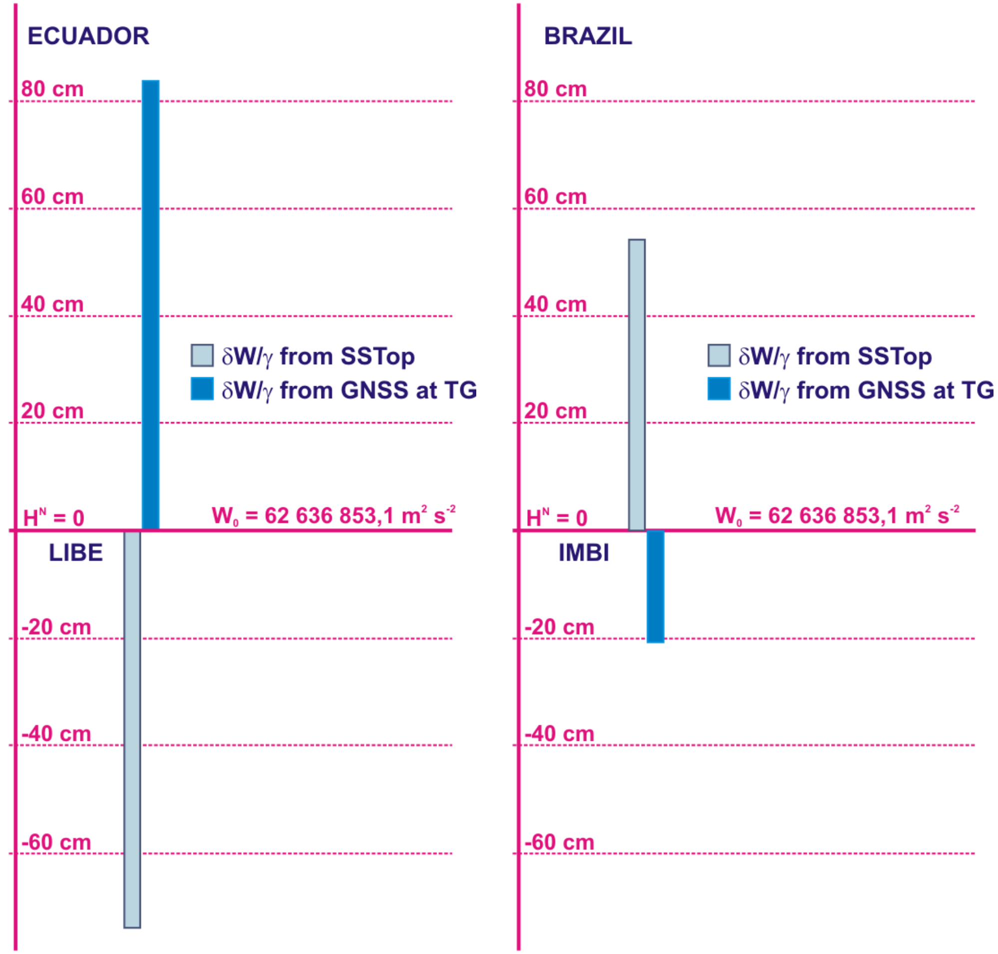

Closing remarks

- ☑ The determination of δW_i must be based on regional geoids of high resolution. The GGMs do not provide the required accuracy and resolution.
- ☑ δW_i terms shall be estimated at the definition period of the local reference levels, i.e. the sea level rise and the vertical crustal movements must be taken into account, and all heights (**h, H^N, ζ, SSTop**) shall be reduced to a reference epoch.
- ☑ The determination of δW_i requires the three proposed approaches: **coastal**, **terrestrial**, and **oceanic approach**. Their isolated evaluation leads to unreliable values.
- ☑ The discussion about introducing orthometric or normal heights should be a **question of the realization, not of the definition**. However, the global vertical system must support both types of heights. In this way, its reference level should be determined where both surfaces (geoid and quasigeoid) are the same: **in oceanic areas**.
- ☑ Although the reference level should be defined by **a fixed W_0 value** (for the computation of geopotential numbers), it must also be realized geometrically by the **(quasi)geoid determination** (solution of the GBVP).
- ☑ The **uniqueness**, **reliability** and **repeatability** of the global reference level **W_0** can be guaranteed by introducing specific **conventions** only, e. g. $V\infty=0$, mean sea surface model, global gravity model, tide system, reference epoch, etc. On the contrary, it will be exist **as many height systems as W_0 computations**.